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ON PARA $\mathcal{G}$-TOPOLOGICAL SIMPLE GROUPS

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Abstract: Para topological group is a group with a topology such that the multiplication mapping is continuous from $G \times G \longrightarrow G$ to G. In this paper we introduce a concept of para generalized topological simple group. Also we $\mathcal{G}$-compactness, $\mathcal{G}$-connected concept for para $\mathcal{G}$-topological simple groups are investigated.

Keywords: para $\mathcal{G}$-topological simple, simple group.

1. INTRODUCTION

Csaszar[5] introduced the notions of generalized topological spaces. He also introduced the notions of associated interior, closure and continuous mappings on generalized neighbourhood system and generalized topological space.

In this paper, we introduce concept of generalized para topological simple group. In[6], C.Selvi, R.Selvi introduced the concept of generalized topological simple groups. This concept is motivated for para generalized topological simple group. It is denoted by para $\mathcal{G}$-topological simple group. Para $\mathcal{G}$-topological simple group has both algebraic and topological structure. Also some basic results and $\mathcal{G}$-compactness, $\mathcal{G}$-connected concept for para $\mathcal{G}$-topological simple groups are introduced and studied.

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2.PRELIMINARIES

Definition: 2.1 [5] Let X be any set and let $\mathcal{G} \subseteq$ P(X) be a subfamily of power set of X. Then $\mathcal{G}$ is called a generalized topology if $\phi \in \mathcal{G}$ and for any index set I, $\bigcup_{i\in I} O_i \in \mathcal{G}$, $O_i \in \mathcal{G}$, i ∈ I.

Definition: 2.2 [5] The elements of $\mathcal{G}$ are called $\mathcal{G}$-open sets. Similarly, generalized closed set (or) $\mathcal{G}$-closed, is defined as complement of a $\mathcal{G}$-open set.

Definition: 2.3 [5] Let X and Y be two $\mathcal{G}$-topological space. A mapping $f: X \to Y$ is called a $\mathcal{G}$-continous on X if for any $\mathcal{G}$-open set O in Y, $f^{-1}(O)$ is $\mathcal{G}$-open in X.

Definition : 2.4 [5] The bijective mapping f is called a $\mathcal{G}$-homeomorphism from X to Y if both f and f^{-1} are $\mathcal{G}$-continous. If there is a $\mathcal{G}$-homeomorphism between X and Y, then they are said to be $\mathcal{G}$-homeomorphic. It is denoted by $X \cong_{\mathcal{G}} Y$.

Definition : 2.5 [5] Collection of all $\mathcal{G}$-interior points of $A \subset X$ is called $\mathcal{G}$-interior of A. It denoted by $Int_{\mathcal{G}}(A)$. By definiton it obvious that $Int_{\mathcal{G}}(A) \subset$ A.

Note: 2.6 [3] *(i)*. $\mathcal{G}$-interior of A, $Int_{\mathcal{G}}(A)$ is equal to union of all $\mathcal{G}$-open sets contained in A.

(ii). $\mathcal{G}$-closure of A as intersection of all $\mathcal{G}$-closed sets containing A. It is denoted by $Cl_{\mathcal{G}}(A)$.

3.ON PARA $\mathcal{G}$-TOPOLOGICAL SIMPLE GROUPS

Definition: 3.1 A para $\mathcal{G}$-topological simple group G is a simple group which is also a $\mathcal{G}$-topological space if the multiplication mapping $m: G \times G \longrightarrow G$ defined by $m(x, y) = x * y$, $x, y \in G$ is $\mathcal{G}$-continuous.

Proposition: 3.2 Let G be a para $\mathcal{G}$-topological simple group. Then the right(left) translation $L_g(R_g)$ of G by g is a $\mathcal{G}$-homeomorphism of the space G onto itself.

Proof: *(i)*.First we will prove that L_g is a bijection. Assume that $y \in G$, then the element $g^{-1}y$ maps to y. So L_g is surjective. Take $L_g(x) = L_g(y)$. Then $gx = gy$. This implies that $x = y$. Therefore L_g is 1-1. Now we shall prove that L_g is a $\mathcal{G}$-homeomorphism. Since $L_g: G \to G$ is equal to the composition,

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$$G \xrightarrow{i_g} G \times G \xrightarrow{m} G, where\ i_g(x) = (g, x), x \in G.$$

Now we want to verify that the map $i_g: G \to G \times G$ is $\mathcal{G}$-continuous. Take any $\mathcal{G}$-open set $U \times V, where\ U, V$ are $\mathcal{G}$-open set in G,

$$i_g^{-1}(U \times V) = \begin{cases} V\ if\ g \in U \\ \varphi\ if\ g \notin U \end{cases}$$

We know that any $\mathcal{G}$-open set in the product $\mathcal{G}$-topology of $G \times G$ can be written as union of $\mathcal{G}$-open sets of the form $U \times V$. Then i_g is $\mathcal{G}$-continuous. We know that $(L_g)^{-1} = L_g^{-1}$ is $\mathcal{G}$-continuous. Therefore the left translation map $L_g: G \longrightarrow G$ is $\mathcal{G}$-continuous. Similarly we will prove for right translation mapping(R_g).

Note: 3.3 Since any two points $g, g' \in G$, there exists a $\mathcal{G}$-homeomorphism $L_{g'g^{-1}}(g) = g'$, any para $\mathcal{G}$-topological simple group is a $\mathcal{G}$-homogeneous space.

Corollary: 3.4 Let G be a para $\mathcal{G}$-topological simple group and U be a $\mathcal{G}$-open subset of G , F is closed in G and g be any element of G. Then

(i). gU and Ug are $\mathcal{G}$-open in G.

(ii). aF and Fa are $\mathcal{G}$-closed in G.

Proof: *(i)*. By theorem 2.2.2, L_g and R_g are $\mathcal{G}$-open map, $L_g(U) = gU$ and $R_g(U) = Ug$ are $\mathcal{G}$-open.

(ii). By the theorem 2.2.2, L_g and R_g are $\mathcal{G}$-closed map, $L_g(U) = gU$ and $R_g(U) = Ug$ are $\mathcal{G}$-closed.

Corollary: 3.5 Let G be a para $\mathcal{G}$-topological simple group and μ_e be a collection of all $\mathcal{G}$-open sets of G at e. Then $\mu_g = \{ Ug: U \in \mu_e\}$ is also a collection of $\mathcal{G}$-open sets at g.

Proof: By above corollary 2.2.4, the proof of the statement is trivially true.

Theorem: 3.6 Let G be a para $\mathcal{G}$-topological simple group, U an $\mathcal{G}$-open subset of G and A be any subset of G. Then AU(respectively, UA) is $\mathcal{G}$-open in G.

Proof: Let g be any element in G. By corollary 2.2.4**,** gU and Ug are $\mathcal{G}$-open in G. Then $AU = \bigcup_{g\in A} gU$ is $\mathcal{G}$-open in G. Similarly UA is $\mathcal{G}$-open in G.

Theorem: 3.7 Let G be a para $\mathcal{G}$-topological simple group and μ_e be the collection of all $\mathcal{G}$-open neighbourhoods of e. Then for every $U \in \mu_e$, For every $g \in U$, there is an element $V \in \mu_e$ such that $V * g \subset U$ and $g * V \subset U$.

Proof: By theorem 2.2.8, the translation mappings are $\mathcal{G}$-homeomorphism, for each $\mathcal{G}$-open set U containing g, there exists a $\mathcal{G}$-open set V at the identity e such that $L_g(V) = g * V \subset U$. Similarly $R_g(V) = V * g \subset U$.

Proposition: 3.8 Every para $\mathcal{G}$-topological simple group G has $\mathcal{G}$-open neighborhood at the identity element e_G consisting of symmetric $\mathcal{G}$-neighbourhoods.

Proof: For an arbitrary $\mathcal{G}$-open neighbourhood U of the identity e_G, if $V = U \cap U^{-1}$, then $V = V^{-1}$, the set V is an $\mathcal{G}$-open neighbourhood of e_G , which implies that V is a symmetric $\mathcal{G}$-neighbourhood and $V \subset U$.

Proposition: 3.9 Let G be a para $\mathcal{G}$-topological simple group. Every $\mathcal{G}$-neighbourhood U of e contains an $\mathcal{G}$-open symmetric neighbourhood V of e such that $VV \subset U$.

Proof: Let U' be the interior of U. Consider the multiplication mapping $\mu: U' \times U' \to G$. Since μ is $\mathcal{G}$-continous, $\mu^{-1}(U')$ is $\mathcal{G}$-open and contains (e, e). Hence there are $\mathcal{G}$-open sets $V_1, V_2 \subset U$ such that $(e, e) \in V_1 \times V_2$, and $V_1V_2 \subset U$. If we let $V_3 = V_1 \cap V_2$, then $V_3V_3 \subset U$ and V_3 is an $\mathcal{G}$-open neighbourhood of e. Finally, let $V = V_3 \cap V_3^{-1}$, which is $\mathcal{G}$-open, contains e and V is symmetric and satisfies $VV \subset U$.

Corollary: 3.10 Let G be a para $\mathcal{G}$-topological simple group. Every $\mathcal{G}$-neighbourhood U of e contains an $\mathcal{G}$-open symmetric neighbourhood V of e such that $V^{-1}V \subset U$ and $VV^{-1} \subset U$.

Proof: Since V is a symmetric $\mathcal{G}$-open neighbourhood, $V = V^{-1}$. Therefore $V^{-1}V \subset U$ and $VV^{-1} \subset U$.

Proposition: 3.11 Suppose that a subgroup of a para $\mathcal{G}$-topological simple group G contains a non-empty $\mathcal{G}$-open subset of G. Then H is $\mathcal{G}$-open in G.

Proof: Let U be a $\mathcal{G}$-open non-empty subset of G with $U \subset H$. For every $g \in H$, By corollary 2.2.4, the set $L_g(U) = gU$ is $\mathcal{G}$-open in G, then $H = \cup_{g\in H}\, gU$ is $\mathcal{G}$-open in G.

Proposition: 3.12 Every $\mathcal{G}$-open subgroup H of a para $\mathcal{G}$-topological simple group G is $\mathcal{G}$-closed in $G\cdot$

Proof: The family $\gamma = \{ Ha : a \in G \}$ of all right cosets of H in G is a disjoint $\mathcal{G}$-open covering of G. Therefore every element of γ, the complement of the union of all other elements, is $\mathcal{G}$-closed in G.

Proposition: 3.13 Let $f: G \to H$ be a homomorphism of para $\mathcal{G}$-topological simple groups.If f is $\mathcal{G}$-continuous at the neutral element e_G of G, then f is $\mathcal{G}$-continuous.

Proof: Let $x \in G$ be arbitrary and suppose that W is an $\mathcal{G}$-open neighbourhood of $y = f(x)$ in H. Since the left translation L_y is a $\mathcal{G}$-homeomorphism of H, there exists an $\mathcal{G}$-open neighbourhood V of the neutral element e_H in H such that $L_y(V) = yV$ is an $\mathcal{G}$-open neighbourhood of y. Then $yV \subseteq W$. Since f is $\mathcal{G}$-continous at e_G of G, then $f(U) \subset V$, for some $\mathcal{G}$-open neighbourhood U of e_G in G. since L_x is a $\mathcal{G}$-homeomorphism of G onto itself, then xU is an $\mathcal{G}$-open neighbourhood of x in G.

Now we have $f(xU) = f(x)f(U)$

$$= y\, f(U)$$

$\subseteq yU$

$\subseteq W$. Hence f is $\mathcal{G}$-continous at the point $x \in G$.

Proposition: 3.14 $SL_n(F)$ is an $\mathcal{G}$-open subset of $M_n(F)$.

Proof: Let A be an element of $M_n(F)$ and let $f(A) = \det(A)$ be a function from $M_n(F)$ to F. Since $SL_n(F)$ contains the matrices of $M_n(F)$ with determinant 1.Then $SL_n(F) = M_n(F)\backslash B$, where B is a matrix with $\det(B) \neq 1$. Since the determinant function is a polynomials and polynomials are $\mathcal{G}$-continous, f is $\mathcal{G}$-continous. Since $\{0\}$ is $\mathcal{G}$-closed in F, then $f^{-1}(\{0\})$ is also $\mathcal{G}$-closed in $M_n(F)$. Hence $SL_n(F)$ is $\mathcal{G}$-open.

Proposition: 3.15 Suppose that G, H and K are para $\mathcal{G}$-topological simple groups and that $\phi: G \to H$ and $\psi: G \to K$ are homomorphism Such that $\psi(G) = K$ and $Ker\ \psi \subset Ker\ \phi$. Then there exists homomorphism $f: K \to H$ such that $\phi = f\ o\ \psi$. In addition, for each $\mathcal{G}$-neighbourhood U of the identity element e_H in H, there exists a $\mathcal{G}$-neighbouhood V of the identity element e_k in K such that $\psi^{-1}(\mathrm{V}) \subset \phi^{-1}(\mathrm{U})$, then f is $\mathcal{G}$-continuous.

Proof: Suppose U is $\mathcal{G}$-neighbourhood of e_H in H. By assumption, there exists a $\mathcal{G}$-neighbouhood V of the identity element e_k in K such that , $W = \psi^{-1}(\mathrm{V}) \subset \phi^{-1}(\mathrm{U})$.

$\Rightarrow \phi(W) = \varphi(\psi^{-1}(\mathrm{V})) \subset \phi(\phi^{-1}(\mathrm{U}))$

$\Rightarrow \phi(W) = f(V) \subset \mathrm{U}$

$\Rightarrow f(V) \subset$ U. Hence f is $\mathcal{G}$-continous at the identity element of K. Therefore f is $\mathcal{G}$-continous.

Corollary: 3.16 Let $\phi: G \to H$ and $\psi: G \to K$ be a $\mathcal{G}$-continous homomorphism of a para $\mathcal{G}$-topological simple groups G, H and K Such that $\psi(G) = K$ and $Ker\ \psi \subset Ker\ \phi$. If the homomorphism ψ is $\mathcal{G}$-open, then there exists a $\mathcal{G}$-continous homomorhism, $f: K \rightarrow H$ such that $\phi = f \ o \ \psi$.

Proof: The existence of a homomorphism $f: K \rightarrow H$ such that $\phi = f \ o \ \psi$. Take an arbitrary $\mathcal{G}$-open set V in H. Then $f^{-1}(V) = \psi(\phi^{-1}(V))$. Since ϕ is $\mathcal{G}$-continous and ψ is an $\mathcal{G}$-open map, $f^{-1}(V)$ is $\mathcal{G}$-open in K. Therefore f is $\mathcal{G}$-continuous.

Proposition: 3.17 Let G and H be two para $\mathcal{G}$-topological simple groups with neutral element e_G and e_H, respectively, and let p be a $\mathcal{G}$-continuous homomorphism of G onto H such that, for some non-empty subset U of G, the set p(U) is $\mathcal{G}$-open in H and the restriction of p to U is an $\mathcal{G}$-open mapping of U onto $p(U)$. Then the homomorphism p is $\mathcal{G}$-open.

Proof: It suffices to show that $x \in G$, where W is an $\mathcal{G}$-open neighbourhood of x in G, then $P(W)$ is a $\mathcal{G}$-open neighbourhood of $p(x)$ in H. Fix a point y in U, and let L be the left translation of G by yx^{-1}. Then L is a $\mathcal{G}$-homeomorphism of G onto itself such that ,

$$L_{yx^{-1}}(x) = yx^{-1}x$$

$$= y.$$

So $V = U \cap L(W)$ is an $\mathcal{G}$-open neighbourhood of y in U. Then $p(V)$ is $\mathcal{G}$-open subset of H. consider the left translation h of H by the inverse to $p(\mathrm{y}x^{-1})$.

Now clearly, $(h \ o \ p \ o \ L)(x) = h(p(L(x)))$

$$= h(p(y))$$

$$= p(xy^{-1})p(y)$$

$$= p(xy^{-1}y)$$

$$= p(x).$$

Hence $h\left(p\big(L(W)\big)\right) = p(W)$. Clearly h is a $\mathcal{G}$-homeomorphism of H onto itself. Since $p(V)$ is $\mathcal{G}$-open in H, $h(p(V))$ is also $\mathcal{G}$-open in H. Therefore $p(W)$ contains the $\mathcal{G}$-open neighbourhood $h(p(V))$ of $p(x)$ in H. Hence $p(W)$ is a $\mathcal{G}$-open neighbourhood of $p(x)$ in H.

Theorem: 3.18 Let $p: G \to H$ be a $\mathcal{G}$-continuous homomorphism of a para $\mathcal{G}$-topological simple groups. Suppose that the image $p(U)$ contains a non-empty $\mathcal{G}$-open set in H, for each $\mathcal{G}$-open neighbourhood U of the neutral element e_G in G. Then the homomorphism p is $\mathcal{G}$-open.

Proof: First we claim that the neutral element e_H of H is in the $\mathcal{G}$-interior of $p(U)$, for each $\mathcal{G}$-open neighbourhood U of e_G in G. Choose an $\mathcal{G}$-open neighbourhood V of e_G such that $V^{-1}V \subset U$. By our assumption, $p(V)$ contains a non empty $\mathcal{G}$-open set W in H. Then $W^{-1}W$ is an $\mathcal{G}$-open neighbourhood of e_H and we have that $W^{-1}W \subset p(V)^{-1}p(V)$

$$= p(V^{-1}V)$$

$$\subset p(U)\ .$$

Choose an arbitrary element $y \in p(U)$, where U is an arbitrary non-empty $\mathcal{G}$-open set in G. We can find $x \in U$ with $p(x) = y$ and $\mathcal{G}$-open neighbourhood V of e_G in G such that $xV \subset U$. Let W be an $\mathcal{G}$-open neighbourhood of e_H with $W \subset p(V)$. Then the set yW contains y, it is $\mathcal{G}$-open in H and ,

$$yW \subset p(xV)$$

$$\subset p(U).$$

This implies that $p(U)$ is $\mathcal{G}$-open in H.

Proposition: 3.19 Let G be a para $\mathcal{G}$-topological simple group. If H is a normal subgroup of G, then $\overline{H}$ also a normal subgroup of G.

Proof: Now we have to prove that $g\overline{H}g^{-1} \in \overline{H}\ \forall\ g \in G$.

Since H is a normal subgroup of G, $gHg^{-1} \in H \ \forall \ g \in G$.

Now $\overline{gHg^{-1}} \subset \overline{H} \ \forall \ g \in G$.

$\Rightarrow g\overline{H}g^{-1} \subset \overline{H} \ \forall g \in G$.

$\Rightarrow g\overline{H}g^{-1} \in \overline{H}, \forall g \in G$. Therefore $\overline{H}$ is a normal subgroup of G.

Corollary: 3.20 Let G be a para $\mathcal{G}$-topological simple group and H be the centre of a hausdorff $\mathcal{G}$-topological simple group G. Then $\overline{H}$ is a subgroup of G.

Proof: Proof follows from the above theorem 3.2.19.

Corollary: 3.21 Let G be a para $\mathcal{G}$-topological simple group and $Z(G)$ be the centre of G. Then $\overline{Z(G)}$ is a normal subgroup of G.

Proof: Proof follows from the above theorem 3.2.19.

Corollary: 3.22 Let G and H be two para $\mathcal{G}$-topological simple groups. If $f: G \rightarrow H$ is a homomorphism mapping , then $\overline{kerf}$ is a normal subgroup of G.

Proof: Proof follows from theorem 3.2.19.

4. QUOTIENTS ON A $\mathcal{G}$-TOPOLOGICAL SIMPLE GROUPS

Let G be a para $\mathcal{G}$-topological simple group and H be a normal subgroup of G. Here H is either proper trivial normal subgroup of G or improper trivial normal subgroup of G. Let ϕ be a mapping from $G \rightarrow \frac{G}{H}$ by $\phi(x) = xH, \forall \ x \in G$. Now we can define a $\mathcal{G}$-topology on $\frac{G}{H}$, U is $\mathcal{G}$-open in $\frac{G}{H} \Leftrightarrow \phi^{-1}(U)$ is $\mathcal{G}$-open in G.

Theorem: 4.1 Let $\frac{G}{H}$ be a para $\mathcal{G}$-topological simple group with quotient $\mathcal{G}$-topology and $\phi : G \rightarrow \frac{G}{H}$ by $\phi(x) = xH, \forall x \in G$. Then the following statement is hold.

(i). ϕ is onto.

(ii). ϕ is $\mathcal{G}$-continous.

(iii). ϕ is $\mathcal{G}$-open.

(iv). ϕ is homomorphism.

Proof: (i). Let $\in \frac{G}{H}$. Since $\phi(x) = gH, \forall g \in G$. Hence ϕ is onto.

(ii). By definition of quotient $\mathcal{G}$-topology, U is $\mathcal{G}$-open in $\frac{G}{H} \Leftrightarrow \phi^{-1}(U)$ is $\mathcal{G}$-open in G. Hence ϕ is $\mathcal{G}$-continous.

(iii). Let U be a $\mathcal{G}$-open set in G. Then $\phi(U) = UH$ is $\mathcal{G}$-open in $\frac{G}{H}$.

(iv). Let $x, y \in G, \phi(xy) = xyH$.

$= xHyH$

$= \phi(x)\phi(y)$.

Hence ϕ is a homomorhism.

5. $\mathcal{G}$-CONNECTEDNESS IN PARA $\mathcal{G}$-TOPOLOGICAL SIMPLE GROUPS

In this section we introduced a $\mathcal{G}$-Connectedness for para $\mathcal{G}$-topological simple groups.

Theorem: 5.1 For any two $\mathcal{G}$-connected subsets E and F of a para $\mathcal{G}$-topological simple group G, their product EF in G is a $\mathcal{G}$-connected subspace of G.

Proof: Since the multiplication is $\mathcal{G}$-continous, the subspace EF of G is a $\mathcal{G}$-continous image of the catesian product $E \times F$ of the spaces E and F. Since $E \times F$ is $\mathcal{G}$-connected, the space is $\mathcal{G}$-connected.

Theorem: 5.2 Let U be an arbitrary $\mathcal{G}$-open neighbourhood of the neutral element e of a $\mathcal{G}$-connected simple topological group G. Then $G = \bigcup_{n=1}^{\infty} U^n$.

Proof: Choose a $\mathcal{G}$-open symmetric neighbourhood V of e in G such that $V \subset U$. By induction on n and by theorem 3.2.6, for every positive integer n, V^n is $\mathcal{G}$-open. Let $H = \bigcup_{n=1}^{\infty} V^n$. Hence $H = \bigcup_{n=1}^{\infty} V^n$ is $\mathcal{G}$-open. Let. Now let

$x \in V^p, y \in V^q$ are the elements from H. Then ,

$x * y \in V^{p+q} \subset H$

$\Longrightarrow x * y \in H.$

Take an element $x \in H$ and $x \in V^k$. Then $x^{-1} \in (V^{-1})^k = V^k \in H$. Therefore H is a subgroup of G. By lemma 3.2.12, H is also a $\mathcal{G}$-closed in G. Since G is $\mathcal{G}$-connected, and H is non empty and both $\mathcal{G}$-closed and $\mathcal{G}$-open, $G = H$. As $V \subset U$, it follows that $G = \bigcup_{n=1}^{\infty} U^n$.

Theorem: 5.3 Let G be a $\mathcal{G}$-connected $\mathcal{G}$-topological simple group and e its identity element. If U is an $\mathcal{G}$-open neighbourhood of e, then G is generated by U.

Proof: Let U be a $\mathcal{G}$-open neighbourhood of e. For each $n \in \mathbb{N}$, we denote by U^n the set of elements of the form $u_1 u_2 \dots u_n$, where $u_i \in U$. Let $= \bigcup_{n \in \mathbb{N}} U^n$. Since each U^n is $\mathcal{G}$-open, we have that W is a $\mathcal{G}$-open, W is also a $\mathcal{G}$-closed by lemma 3.2.12. Let g be an element of generalized closure of W. That is $g \in Cl_{\mathcal{G}}(W)$. Since gU^{-1} is a $\mathcal{G}$-open neighbourhood of g, it must intersects W. Thus, let $h \in W \cap gU^{-1}$. Since $h \in gU^{-1}$, then $h = gu^{-1}$ for some elements $u \in U$. Since $h \in W$, then $h \in U^n$ for some $n \in \mathbb{N}$. So $h = u_1 u_2 \dots u_n$ with each $u_i \in U$.

We have $g = hu$ for some $u \in U$.

$= u_1 u_2 \dots u_n u$ for some $u \in U$.

This implies that $g \in U^{n+1} \subseteq W$. Hence W is $\mathcal{G}$-closed. Since G is $\mathcal{G}$-connected and W is $\mathcal{G}$-open and $\mathcal{G}$-closed, we must have $W = G$. This means that G is generated by U.

Theorem: 5.4 Let K be a discrete invariant subgroup of a $\mathcal{G}$-connected para $\mathcal{G}$-topological simple abelian group G. If for any $\mathcal{G}$-open neighbourhood U of x in G there exists a $\mathcal{G}$-open symmetric neighbourhood V of e in G such that $VxV \subset U$, then every element of K commutes with every element of G. i.e. K is contained in the center of the simple abelian group G.

Proof: Since G is a $\mathcal{G}$-topological simple abelian group, G and $\{e\}$ are the only invariant subgroup of G. Therefore the proof is trivial.

Theorem: 5.5 Let G be a para $\mathcal{G}$-topological simple group and H is a $\mathcal{G}$-connected subgroup of G. Then

(i). If $\frac{G}{H}$ is $\mathcal{G}$-connected, then G is $\mathcal{G}$-connected.

(ii). If H is a $\mathcal{G}$-closed invariant subgroup of G and $\frac{G}{H}$ is $\mathcal{G}$-connected, then G is $\mathcal{G}$-connected.

(iii). If H is a $\mathcal{G}$-dense subgroup of a $\mathcal{G}$-connected $\mathcal{G}$-topological simple group, then every $\mathcal{G}$-neighbourhood U of the identity element in H algebraically generates the group H.

Proof: *(i).* Suppose G is not $\mathcal{G}$-connected. Then there exists two disjoint non empty $\mathcal{G}$-open set U, V such that $U \cup V = G$. Denoting by π the map $x \to xH$, we see that $\frac{G}{H} = \pi(U) \cup \pi(V)$. By theorem 3.3.1 , π is a $\mathcal{G}$-open mapping and $\frac{G}{H}$ is the union of two nonempty $\mathcal{G}$-open sets $\pi(U)$ and $\pi(V)$. Now the assumption of $\mathcal{G}$-connectedness of $\frac{G}{H}$ implies that $\pi(U)$ and $\pi(V)$ have some common point, zH. But disjointness assumption on U and V means that, for this to happen there must exists $x \in$ U and $y \in V$ such that $x^{-1}y \in H$. So the coset xH intersects both U and V in G. As $U \cup V = G$ provides a disconnection xH has to be disconnected. But xH is homeomorphic to H. So this contradicts the hypothesis that H is $\mathcal{G}$-connected. So G is $\mathcal{G}$-connected.

(ii) . Suppose that H and $\frac{G}{H}$ are $\mathcal{G}$-connected and $f: G \to \{0,1\}$ be an arbitrary $\mathcal{G}$-continous map. The restriction of f to H must be constant and since each coset gH is $\mathcal{G}$-connected, f must be constant on gH as well taking value $f(g)$. Thus we have a well defined map $\tilde{f}: \frac{G}{H} \to \{0,1\}$ such that $\tilde{f}\ o\ \pi = f$. By the fundamental property of quotient spaces, it follows that $\tilde{f}$ is $\mathcal{G}$-continous and so must be constant. Since $\frac{G}{H}$ is $\mathcal{G}$-connected. Hence, f is also constant and we conclude that G is $\mathcal{G}$-connected.

(iv). Proof follows from theorem 3.4.3

6. $\mathcal{G}$- COMPACTNESS ON PARA $\mathcal{G}$-TOPOLOGICAL SIMPLE GROUP

Proposition: 6.1 If G is a para $\mathcal{G}$-topological simple group and if K_1 and K_2 are $\mathcal{G}$-compact subsets of G, then K_1K_2 is $\mathcal{G}$-compact.

Proof: Since K_1, K_2 are $\mathcal{G}$-compact subsets of G, then $K_1 \times K_2$ is $\mathcal{G}$-compact in $G \times G$. since the $\mathcal{G}$-continous image of a compact set is compact, K_1K_2 is $\mathcal{G}$-compact.

Definition: 6.2 Let H be a subgroup of a para $\mathcal{G}$-topological simple group G. Then H is called neutral in G if every $\mathcal{G}$-neighbourhood U of the identity e_G in G, there exists a $\mathcal{G}$-neighbourhood V of e_G such that $VH \subset HU$.

Proposition: 6.3 Let H be a subgroup of a para $\mathcal{G}$-topological simple group G. Suppose that, for every $\mathcal{G}$-open neighbourhood U of the identity e_G in G, there exists an $\mathcal{G}$-open neighbourhood V of e_G in G such that $xVx^{-1} \subset U$ whenever $x \in G$. Then H is neutral in G.

Proof: Given a $\mathcal{G}$-neighbourhood U of e_G in G. Take an $\mathcal{G}$-open neighbourhood V of e_G satisfying,

$xVx^{-1} \subset U, \forall\ x \in G$

$\Rightarrow xV \ \subset Ux, \forall\ x \in G$

$\Rightarrow HV \subset UH, \forall x \in G$. Then H is neutral in G.

Corollary: 6.4 Every $\mathcal{G}$-compact subgroup of a para $\mathcal{G}$-topological simple group G is neutral in G.

Proof: Proof follows from the theorem

Proposition: 6.5 Let G be a para $\mathcal{G}$-topological simple group, F a $\mathcal{G}$-closed subset of G, and K is a $\mathcal{G}$-compact subset of G, such that $F \cap K = \phi$. Then there is an $\mathcal{G}$-open neighbourhood V of e_G such that $F \cap VK = \phi$.

Proof: Let $x \in K$.

$\Rightarrow x \in G\backslash F$ and $G\backslash F$ is $\mathcal{G}$-open.

$\Rightarrow (G\backslash F)x^{-1}$ is an $\mathcal{G}$-open neighbourhood of e_G. By Corollary: 3.2.4,

there is an $\mathcal{G}$-open neighbourhood W_x of e_G such that $W_x W_x \subset (G\backslash F)x^{-1}$. Since K is $\mathcal{G}$-compact and $K \subset \bigcup_{x\in K} W_x x$, so there are finite points $x_1, x_2, \ldots, x_n \in K$, such that

$K \subset \cup_{i=1}^{n} W_i x_i$, where $W_i = W_{x_i}$ .Now let $V = \cap_{i=1}^{n} W_i$. For any $x \in K, x \in W_i x_i$,for some i. This implies that,

$Vx \subset W_i x \subset W_i W_i x_i \subset G\backslash F$.

$\Rightarrow F \cap Vx = \phi$. This true for any $x \in K$, therefore $F \cap VK = \phi$.

Proposition: 6.6 Let G be a para $\mathcal{G}$-topological simple group, K a $\mathcal{G}$-compact subset of G, and F is a $\mathcal{G}$-closed subset of G. Then FK and KF are $\mathcal{G}$-closed subsets of G.

Proof: If $FK = G$, this is obviously true. Now let $y \in G\backslash FK$. Then $F \cap y K^{-1} = \phi$. Since K is $\mathcal{G}$-compact, $y K^{-1}$ is $\mathcal{G}$-compact. By above theorem, there is an $\mathcal{G}$-open neighbourhood V of e_G

such that $F \cap Vy\ K^{-1} = \phi$. Since Vy is an $\mathcal{G}$-open neighbourhood of y contained in $G\backslash FK$. Therefore FK is $\mathcal{G}$-closed.

Proposition: 6.7 Let G be a $\mathcal{G}$-compact $\mathcal{G}$-topological simple group and H be a $\mathcal{G}$-topological simple group and let $f: G \to H$ be $\mathcal{G}$-continous. Then $f(G)$ is $\mathcal{G}$-compact.

Proof: Let γ be an $\mathcal{G}$-open cover for $f(G)$**.** We know that $\{f^{-1}(U): U \in \gamma\}$ is an $\mathcal{G}$-open cover of G. Since G is $\mathcal{G}$-compact, there is a finite $\mathcal{G}$-subcover. There exists $U_1, U_2, \dots, U_n \in \gamma$ such that

$G \subseteq f^{-1}(U_1) \cup f^{-1}(U_2) \cup \dots \cup f^{-1}(U_n)$

$\Rightarrow f(G) \subseteq U_1 \cup U_2 \cup \dots \cup U_n$. Thus an arbitrary $\mathcal{G}$-open cover of $f(G)$ has a finite $\mathcal{G}$-subcover.

Proposition: 6.8 Let G be a $\mathcal{G}$-compact para $\mathcal{G}$-topological simple group and let H be $\mathcal{G}$-closed subset of G. Then H is $\mathcal{G}$-compact.

Proof: Let γ be an $\mathcal{G}$-open cover of H. Since H is $\mathcal{G}$-closed in G, $G\backslash H$ is $\mathcal{G}$-open in G. From this we get that $\gamma \cup G\backslash H$ is an $\mathcal{G}$-open cover for G. Since G is $\mathcal{G}$-compact, there must be a finite $\mathcal{G}$-subcover. So there exists $U_1, U_{2,} \dots, U_n \in \gamma$, such that $G \subset U_1 \cup U_2 \cup \dots \cup U_n \cup G\backslash H$

$\Rightarrow H \subset U_1 \cup U_2 \cup \dots \cup U_n$ **.** An arbitrary $\mathcal{G}$-open cover of H has a finite $\mathcal{G}$-subcover. Therefore H is $\mathcal{G}$-compact.

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